OCR
Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics A
Y535 Additional Pure Mathematics
Sample Question Paper

MODEL ANSWERS

Date – Morning/Afternoon
Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Book
- Formulae AS Level Further Mathematics A

You must have:
- Printed Answer Book
- Formulae AS Level Further Mathematics A
- Scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

1 The sequence $\{u_n\}$ is defined by $u_1 = 2$ and $u_{n+1} = \dfrac{12}{1+u_n}$ for $n \geq 1$.

Given that the sequence converges, with limit α, determine the value of α. **[3]**

sequence converges $\Rightarrow$ nth & (n+1)th terms are equal as $n \to \infty$

$$U_{n+1} = U_n = \alpha \Rightarrow \alpha = \frac{12}{1+\alpha}$$

$$\alpha^2 + \alpha - 12 = 0$$

$$\Rightarrow \alpha = 3 \text{ or } \alpha = -4$$

1^{st} term is +ve $\Rightarrow$ all terms are +ve

$$\therefore \alpha > 0 \Rightarrow \alpha = 3$$

2 The points A (1, 2, 2), B (8, 2, 5), C (–3, 6, 5) and D (–10 , 6, 2) are the vertices of parallelogram $ABCD$.

Determine the area of $ABCD$. **[5]**

$$\text{area} = |(\underline{b}-\underline{a}) \times (\underline{c}-\underline{a})|$$

$$\underline{b}-\underline{a} = \begin{pmatrix} 8 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix} \qquad \underline{c}-\underline{a} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix}$$

$$\left| \begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix} \right| = \left| \begin{pmatrix} -12 \\ -33 \\ 28 \end{pmatrix} \right|$$

$$\sqrt{12^2 + 33^2 + 28^2} = \sqrt{2017}$$
$$= 44.9$$

3 A non-commutative group G consists of the six elements $\{e, a, a^2, b, ab, ba\}$ where e is the identity element, a is an element of order 3 and b is an element of order 2.
By considering the row in G's group table in which each of the above elements is pre-multiplied by b, show that $ba^2 = ab$. **[5]**

$$bG = \{be, ba, ba^2, b^2, bab, b^2a\}$$

$$b^2 = e \Rightarrow bG = \{b, ba, ?, e, ?, a\}$$

Latin-square

$$\therefore ba^2 = a^2 \text{ OR } ab$$

ba^2 can only $= a^2$ if $b = e$

BUT $b \neq e \Rightarrow ba^2 = ab$

4 Let S be the set $\{16, 36, 56, 76, 96\}$ and $\times_H$ the operation of multiplication modulo 100.

(i) Given that a and b are odd positive integers, show that $(10a+6)(10b+6)$ can also be written in the form $10n+6$ for some odd positive integer n. **[3]**

(ii) Construct the Cayley table for $(S, \times_H)$ **[2]**

(iii) Show that $(S, \times_H)$ is a group.
[You may use the result that $\times_H$ is associative on S.] **[3]**

(iv) Write down all generators of $(S, \times_H)$. **[1]**

i. $(10a+6)(10b+6) = 100ab + 60(a+b) + 36$
$= 10(10ab + 6(a+b) + 3) + 6$

$\Rightarrow n = 10ab + 6(a+b) + 3$

n is odd $\because n = \text{even} + \text{even} + \text{odd}$

ii.

	16	36	56	76	96
16	56	76	96	16	36
36	76	96	16	36	56
56	96	16	36	56	76
76	16	36	56	76	96
96	36	56	76	96	16

iii. 1. there are no new elements in the table $\therefore$ group is closed

2. identity = 76

3. inverse pairs: 16, 36 & 56, 96

iv. possible generators: 16, 36, 56, 96

5 Let $f(x, y) = x^3 + y^3 - 2xy + 1$. The surface S has equation $z = f(x, y)$.

(i) **(a)** Find f_x. **[1]**

(b) Find f_y. **[1]**

(c) Show that S has a stationary point at (0, 0, 1). **[5]**

(d) Find the coordinates of the second stationary point of S. **[1]**

(ii) The section $z = f(a, y)$, where a is a constant, has exactly one stationary point. Determine the equation of the section. **[3]**

i. a) $f_x = 3x^2 - 2y$

b) $f_y = 3y^2 - 2x$

c) set f_x & f_y to 0: $3x^2 - 2y = 0$, $3y^2 - 2x = 0$

$\Rightarrow y = \frac{3}{2}x^2$ ① $\quad y^2 = \frac{2}{3}x$ ②

sub ① in ②: $\frac{9}{4}x^4 = \frac{2}{3}x$

$\Rightarrow 27x^4 - 8x = 0$

$\Rightarrow x = 0$ or $x = \frac{2}{3}$

$\therefore$ S has stationary point @ $x = 0$

$\Rightarrow (x, y, z) = (0, 0, 1)$

d) $\left(\frac{2}{3}, \frac{2}{3}, \frac{19}{27}\right)$

ii. @ $x=a$, $f_y = 3y^2 - 2a = 0$

$\Rightarrow y = \pm\sqrt{\frac{3}{2}a}$

$\therefore (0,0,1)$ implies $a=0$

$z = f(a,y) = y^3 - 2ay + 1 + a^3$

$a=0$: $z = y^3 + 1$

6 A customer takes out a loan of £P from a bank at an annual interest rate of 4.9%. Interest is charged monthly at an equivalent monthly interest rate. This interest is added to the outstanding amount of the loan at the end of each month, and then the customer makes a fixed monthly payment of £M in order to reduce the outstanding amount of the loan.

Let L_n denote the outstanding amount of the loan at the end of month n **after** the fixed payment has been made, with $L_0 = P$.

(i) Explain how the outstanding amount of the loan from one month to the next is modelled by the recurrence relation

$$L_{n+1} = 1.004L_n - M \qquad (*)$$

with $L_0 = P$, $n \geq 0$.

[3]

(ii) Solve, in terms of n, M and P, the first order recurrence relation given in part **(i)**. **[6]**

(iii) The loan amount is £100 000 and will be fully repaid after 10 years. Find, to the nearest pound, the value of the monthly repayment. **[3]**

(iv) The bank's procedures only allow for calculations using integer amounts of pounds. When each monthly amount of the outstanding debt (L_n) is calculated it is always rounded **up** to the nearest pound before the monthly repayment (M) is subtracted.
Rewrite (*) to take this into account. **[2]**

i. annual interest $1.049 \rightarrow$ monthly interest $\sqrt[12]{1.049}$

$\sqrt[12]{1.049} = 1.004 = R$

amount owed @ end of month n = $R \times$ amount owed @ end of month $(n-1)$

$-M$: monthly repayment

ii. $L_{n+1} - RL_n = -M$ has complementary solution $L_n = AR^n$

particular solution: trial $L_n = b$

sub into recurrence relation: $b(R-1) = M \Rightarrow b = 250M$

$\therefore$ general solution: $L_n = AR^n + 250M$

$L_0 = P \Rightarrow A = P - 250M$

gives $L_n = (P - 250M) \times 1.004^n + 250M$

iii. $L_n = 0$ when $n = 120$ & $P = 100000$

$$(100000 - 250M) \times 1.004^{120} + 250M = 0$$

$\Rightarrow M = 1051$ to nearest £

iv. take INT of $(1.004L_n + 1)$

so $(*) \rightarrow L_{n+1} = \text{INT}(1.004L_n + 1) - M$

7 **(i)** Let $N = 10a + b$ and $M = a - 5b$ where a and b are integers such that $a \geq 1$ and $0 \leq b \leq 9$. N is to be tested for divisibility by 17.

(a) Prove that $17 \mid N$ if and only if $17 \mid M$. **[5]**

(b) Demonstrate step-by-step how an algorithm based on these forms can be used to show that $17 \mid 4097$. **[2]**

(ii) **(a)** Show that, for $n \geq 2$, any number of the form 1001_n is composite. **[3]**

(b) Given that n is a positive even number, provide a counter-example to show that the statement "any number of the form 10001_n is prime" is false. **[3]**

END OF QUESTION PAPER

i. a) let $17 \mid M$ so $M = 17m$

$a - 5b = 17m$

$N = 10a + b = 10(a - 5b) + 51b$

$= 17m + 51b$

$= 17(m + 3b)$ $\therefore 17 \mid N$

let $17 \mid N$ so $N = 17n$

$10a + b = 17n$

$10M = 10a - 50b = (10a + b) - 51b$

$= 17n - 51b$

$= 17(n - 3b)$

$HCF(10, 17) = 1$ so $17 \mid M$

$\therefore \ 17 \mid N$ iff. $17 \mid M$

b) $4097 \rightarrow a=409, b=7$

$\rightarrow M = 409-35 = 374$

$374 \rightarrow a=37, \ b=4$

$\rightarrow M = 37-20 = 17$

then $17 \mid 17 \Rightarrow 17 \mid 4097$

ii. a) $1001_n = n^3+1 \equiv (n+1)(n^2-n+1)$

$n \geqslant 2 \Rightarrow (n+1) \geqslant 3$ & $n^2-n+1 \geqslant 3$

$\therefore$ neither factor $= 1$

so 1001_n is composite

b) $1001_n = n^4+1$ $\qquad n = 2 \quad 4 \quad 6 \quad 8$

$n^4+1 = 17 \quad 257 \quad 1297 \quad 4097 = 17k$

we know 17 is a factor of $4097 \Rightarrow 4097$ not prime,

& so $n=8$ is a counter-example